Integral of $|x|$

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$$\int_{-1}^{2} |x| \, dx.$$

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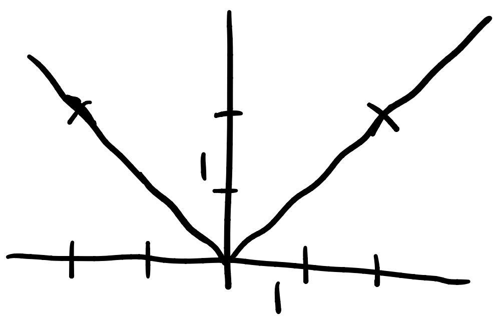

$$\frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$$

$$\text{Area}^{+} = \frac{2}{n}\left(\frac{2}{n}\right) + \frac{2}{n}\left(\frac{2\cdot 2}{n}\right) + \dots + \frac{2}{n}\left(\frac{n\cdot 2}{n}\right)$$

$$= \frac{4}{n^2}\left(1+2+3+\dots+n\right)$$

$$= \frac{4}{n^2}\left(\frac{n}{2}(1+n)\right)$$

$$= \frac{2}{n}(1+n)$$

$$= 2\left(\frac{1}{n}+1\right)$$

$$\text{Area}^{-} = \frac{1}{n}\left(\frac{1}{n}\right) + \frac{1}{n}\left(\frac{2\cdot 1}{n}\right) + \frac{1}{n}\left(\frac{3\cdot 1}{n}\right) + \dots + \frac{1}{n}\left(\frac{n\cdot 1}{n}\right)$$

$$= \frac{1}{n^2}\left(1+2+3+\dots+n\right)$$

$$= \frac{1}{n^2}\left(\frac{n}{2}(1+n)\right)$$

$$= \frac{1}{2n}(1+n)$$

$$= \frac{1}{2}\left(\frac{1}{n}+1\right)$$

$$\lim_{n\to\infty} \text{Area}^{+} + \text{Area}^{-}$$

$$= \lim_{n\to\infty} 2\left(\frac{1}{n}+1\right) + \frac{1}{2}\left(\frac{1}{n}+1\right)$$

$$= 2 + \frac{1}{2}$$

$$= 2\frac{1}{2}$$